Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2012

Marking Scheme

Applied Mathematics

Higher Level

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(–1)
Blunders	- mathematical errors	B(–3)
Misreading	- if not serious	M(–1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

1. **(a)** A particle falls from rest from a point P. When it has fallen a distance 19·6 m a second particle is projected vertically downwards from P with initial velocity $39{\cdot}2\ \text{m s}^{-1}$.
The particles collide at a distance d from P.

Find the value of d.

$$s = ut + \tfrac{1}{2}ft^2$$
$$19{\cdot}6 = 0 + \tfrac{1}{2}gt^2$$
$$t = 2\ \text{s} \qquad 5$$

$$d = 0 + \tfrac{1}{2}g(t+2)^2 \qquad 5$$

$$d = 39{\cdot}2t + \tfrac{1}{2}gt^2 \qquad 5$$

$$\tfrac{1}{2}g(t+2)^2 = 39{\cdot}2t + \tfrac{1}{2}gt^2$$
$$t^2 + 4t + 4 = 8t + t^2$$

$$\Rightarrow \quad t = 1 \qquad 5$$

$$d = 0 + \tfrac{1}{2}g(t+2)^2$$
$$= 4{\cdot}9 \times 9$$
$$= 44{\cdot}1\ \text{m} \qquad 5$$

25

1. **(b)** A car, starts from rest at A, and accelerates uniformly at $1\ \text{m s}^{-2}$ along a straight level road towards B, where $|AB| = 1914$ m. When the car reaches its maximum speed of $32\ \text{m s}^{-1}$, it continues at this speed for the rest of the journey.

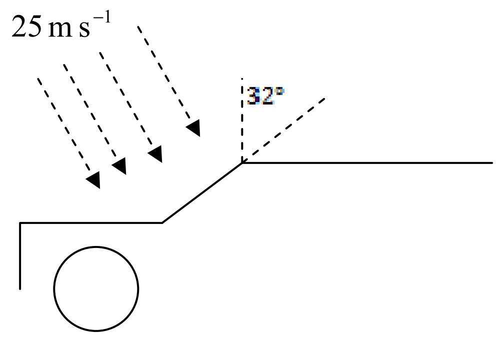
At the same time as the car starts from A a bus passes B travelling towards A with a constant speed of $36\ \text{m s}^{-1}$. Twelve seconds later the bus starts to decelerate uniformly at $0{\cdot}75\ \text{m s}^{-2}$.

(i) The car and the bus meet after t seconds. Find the value of t.

(ii) Find the distance between the car and the bus after 48 seconds.

(i) car $\quad v = u + ft \qquad v^2 = u^2 + 2fs$ $32 = 0 + 1(t) \qquad 32^2 = 0 + 2(1)s$ $t = 32 \qquad s = 512$	5, 5
bus $\quad s = ut + \frac{1}{2}ft^2$ $= 36\times 12 + 0$ $s = 432$	5
$1914 = s_c + s_b$ $= \{512 + 32(t-32)\} + \{432 + 36(t-12) + \frac{1}{2}(-\frac{3}{4})(t-12)^2\}$ $0 = 3t^2 - 616t + 19840$ $\Rightarrow \quad t = 40\ \text{s}$	5
(ii) car $\quad s = ut + \frac{1}{2}ft^2$ $= 32\times 8 + 0$ $= 256$	
bus $\quad v = u + ft \qquad s = ut + \frac{1}{2}ft^2$ $= 36 - \frac{3}{4}(40 - 12) \qquad = 15(8) + \frac{1}{2}(-\frac{3}{4})(64)$ $v = 15 \qquad s = 96$	
$\Rightarrow$ distance $= 256 + 96 = 352$ m	5
	25

2. **(a)** Rain is falling with a speed of $25 \ \text{m s}^{-1}$ at an angle of $20°$ to the vertical.
A car is travelling along a horizontal road into the rain. The windscreen of the car makes an angle of $32°$ with the vertical.
The car is travelling at $20 \ \text{m s}^{-1}$.
Find the angle at which the rain appears to strike the windscreen.



$\vec{V}_r = \ 25\sin 20 \ \vec{i} - 25\cos 20\,\vec{j}$	5	
$\vec{V}_c = \ -20\,\vec{i} + 0\,\vec{j}$	5	
$\vec{V}_{rc} = \ \vec{V}_r - \vec{V}_c$		
$= \ (25\sin 20 + 20\ )\ \vec{i} - 25\cos 20\,\vec{j}$		
$= 28{\cdot}55 \ \ \vec{i} - 23{\cdot}49\,\vec{j}$	5	
$\alpha = \tan^{-1}\left(\dfrac{28{\cdot}55}{23{\cdot}49}\right) + 32$	5	
$= 82{\cdot}55°.$		20

2 **(b)** At noon ship A is 50 km north of ship B.
Ship A is travelling southwest at $24\sqrt{2}$ km h^{-1}. Ship B is travelling due west at 17 km h^{-1}.

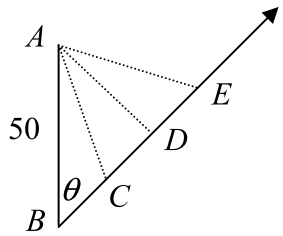
(i) Find the magnitude and direction of the velocity of B relative to A.

A and B can exchange signals when they are not more than 20 km apart.

(ii) At what time can they begin to exchange signals?
(iii) How long can they continue to exchange signals?

(i)
$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$
$$= \left(-17\vec{i} + 0\vec{j}\right) - \left(-24\vec{i} - 24\vec{j}\right)$$
$$= 7\vec{i} + 24\vec{j} \qquad 5$$

$$\left|\vec{V}_{BA}\right| = 25 \qquad \theta = \tan^{-1}\left(\frac{7}{24}\right) \qquad 5$$



(ii)
$$|AD| = 50\sin\theta = 50 \times \frac{7}{25} = 14$$
$$|CD| = \sqrt{20^2 - 14^2} = 2\sqrt{51} = 1{\cdot}2829$$
$$|BC| = 50\cos\theta - 14{\cdot}2829$$
$$= 50 \times \frac{24}{25} - 14{\cdot}2829 = 33{\cdot}7171 \qquad 5$$

$$t_{BC} = \frac{|BC|}{25} = 1{\cdot}3487$$
time = 13:21   5

(iii)
$$|CE| = 2 \times |CD| = 28{\cdot}5658 \qquad 5$$
$$t_{CE} = \frac{|CE|}{25} = 1{\cdot}1426$$
time = 1 h 9 min   5

30

3. **(a)** A particle is projected with a speed of 98 m s^{-1} at an angle α to the horizontal. The range of the particle is 940·8 m. Find

(i) the two values of α

(ii) the difference between the two times of flight.

(i)

$$98\cos\alpha . t = 940{\cdot}8$$ 5

$$t = \frac{9{\cdot}6}{\cos\alpha}$$

$$98\sin\alpha . t - 4{\cdot}9t^2 = 0$$ 5

$$98\sin\alpha - 4{\cdot}9\left(\frac{9{\cdot}6}{\cos\alpha}\right) = 0$$

$$\sin 2\alpha = 0{\cdot}96$$

$$2\alpha = 73{\cdot}74 \quad , \quad 106{\cdot}26$$

$$\alpha = 36{\cdot}87° \quad , \quad 53{\cdot}13°$$ 5

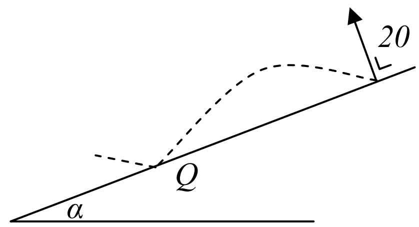
(ii)

$$t_1 = \frac{9.6}{\cos 3{\cdot}87} = 12{\cdot}00$$

$$t_2 = \frac{9{\cdot}6}{\cos 53{\cdot}13} = 16{\cdot}00$$

$$t_2 - t_1 = 4 \text{ s.}$$ 5 | 20

3 **(b)** A particle is projected down an inclined plane with initial velocity 20 m s^{-1}. The line of projection makes an angle of 90° with the inclined plane and the plane is inclined at α to the horizontal. The plane of projection is vertical and contains the line of greatest slope. The particle strikes the plane at Q. The range of the particle on the inclined plane is $\dfrac{1600\sqrt{3}}{g}$. Find



(i) the value of α

(ii) the magnitude of the rebound velocity at Q if the coefficient of restitution is $\frac{1}{2}$.

(i)

$$r_j = 0$$

$$20t - \tfrac{1}{2} g\cos\alpha \times t^2 = 0 \qquad 5$$

$$t = \frac{40}{g\cos\alpha} \qquad 5$$

$$r_i = \frac{1600\sqrt{3}}{g}$$

$$0\times t + \tfrac{1}{2} g\sin\alpha\times t^2 = \frac{1600\sqrt{3}}{g} \qquad 5$$

$$\tfrac{1}{2} g\sin\alpha \times \frac{1600}{g^2\cos^2\alpha} = \frac{1600\sqrt{3}}{g} \qquad 5$$

$$\sin\alpha = 2\sqrt{3}\ \cos^2\alpha$$

$$\Rightarrow \quad \alpha = 60° \qquad 5$$

$$v_i = 0 + g\sin\alpha\times t = g\times\frac{\sqrt{3}}{2}\times\frac{80}{g} = 40\sqrt{3}$$

$$v_j = 20 - g\cos\alpha\times t = 20 - g\times\frac{1}{2}\times\frac{80}{g} = -20$$

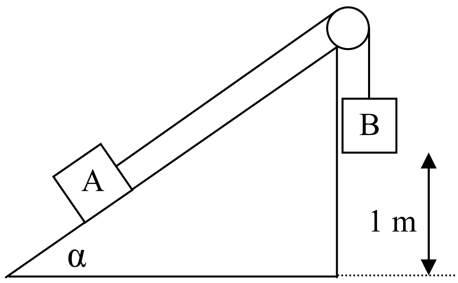
(ii)

$$\left.\begin{aligned} v_i &= 40\sqrt{3} \\ v_j &= 10 \end{aligned}\right\}$$

$$|v| = \sqrt{\left(40\sqrt{3}\right)^2 + (10)^2} \qquad 5 \qquad \boxed{30}$$

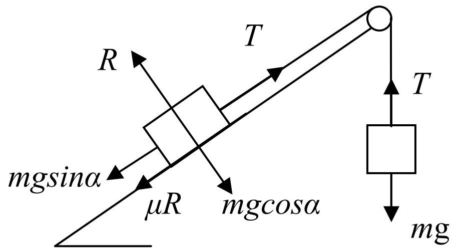
$$|v| = 70 \text{ m s}^{-1}$$

4. **(a)** Two particles A and B each of mass m are connected by a light inextensible string passing over a light, smooth, fixed pulley. Particle A rests on a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{5}{12}$.
Particle B hangs vertically 1 m above the ground.
The coefficient of friction between A and the inclined plane is $\frac{1}{2}$.
The system is released from rest.



(i) Find the speed with which B strikes the ground.

(ii) How far will A travel after B strikes the ground?



(i)

$$mg - T = mf \qquad 5$$

$$T - mg\sin\alpha - \tfrac{1}{2}mg\cos\alpha = mf \qquad 5$$

$$mg - \frac{5mg}{13} - \frac{6mg}{13} = 2mf$$

$$f = \frac{g}{13} \qquad 5$$

$$v^2 = u^2 + 2fs$$

$$= 0 + 2\times\frac{g}{13}\times 1$$

$$v = \sqrt{\frac{2g}{13}} \qquad 5$$

(ii)

$$v^2 = u^2 + 2fs$$

$$0 = \frac{2g}{13} + 2\times\left(\frac{-11g}{13}\right)\times s$$

$$s = \frac{1}{11} \text{ m.} \qquad 5$$

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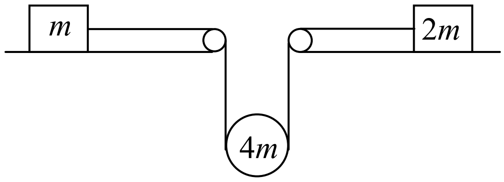
4 **(b)** Two particles of mass m kg and $2m$ kg lie at rest on horizontal rough tables. The coefficient of friction between each particle and the table it lies on is μ $\left(\mu < \frac{2}{3}\right)$.



The particles are connected by a light inextensible string which passes under a smooth movable pulley of mass $4m$ kg.

The system is released from rest.

(i) Find, in terms of m and μ, the tension in the string.

(ii) If the acceleration of the m kg mass is f, find the acceleration of the $2m$ kg mass in terms of f.

(i)

$$T - \mu(mg) = mf \qquad 5$$

$$T - \mu(2mg) = 2mp \qquad 5$$

$$4mg - 2T = 4m \times \tfrac{1}{2}(f + p) \qquad 5$$

$$= 2mf + 2mp$$

$$4mg - 2T = 2T - 2\mu mg + T - 2\mu mg$$

$$T = \frac{4mg(1+\mu)}{5} \qquad 5$$

(ii)

$$mf = T - \mu(mg)$$

$$= \frac{4mg(1+\mu)}{5} - \mu mg$$

$$f = \frac{4g}{5} - \frac{\mu g}{5}$$

$$2mp = T - \mu(2mg)$$

$$2mp = \frac{4mg(1+\mu)}{5} - 2\mu mg$$

$$p = \frac{2g}{5} - \frac{3\mu g}{5}$$

$$p = 3f - 2g \qquad 5$$

25

5. **(a)** Three smooth spheres, A, B and C, of mass $3m$, $2m$ and m lie at rest on a smooth horizontal table with their centres in a straight line. Sphere A is projected towards B with speed 5 m s^{-1}. Sphere A collides directly with B and then B collides directly with C.

The coefficient of restitution between the spheres is e.

Show that if $e > \dfrac{3-\sqrt{5}}{2}$ there will be no further collisions.

PCM	$3m(5)+2m(0) = 3mv_1 + 2mv_2$	5
NEL	$v_1 - v_2 = -e(5-0)$	5
	$v_1 = 3-2e$, $v_2 = 3+3e$	5
PCM	$2m(v_2)+m(0) = 2mv_3 + mv_4$	5
NEL	$v_3 - v_4 = -e(v_2-0)$	

$$v_3 = \frac{v_2}{3}(2-e) = (1+e)(2-e)$$

$$v_4 = \frac{2v_2}{3}(1+e) = 2(1+e)(1+e)$$

$$v_1 < v_3 < v_4$$

$$v_1 < v_3 \text{ if}$$

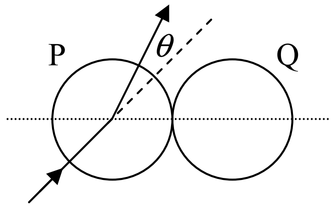
$$3-2e < (1+e)(2-e)$$

$$3-2e < 2+e-e^2$$

$$e^2-3e+1<0 \Rightarrow \quad \frac{3-\sqrt{5}}{2} < e < \frac{3+\sqrt{5}}{2}$$

$$\Rightarrow v_1 < v_3 \quad \text{if } e > \frac{3-\sqrt{5}}{2} \qquad 5 \qquad 25$$

5 **(b)** A smooth sphere P collides with an identical smooth sphere Q which is at rest. The velocity of P before impact makes an angle α with the line of centres at impact, where $0° \le \alpha < 90°$.



The velocity of P is deflected through an angle θ by the collision.

The coefficient of restitution between the spheres is $\frac{1}{3}$.

Show that $\tan\theta = \dfrac{2\tan\alpha}{1+3\tan^2\alpha}$.

PCM $\quad m(u\cos\alpha) + m(0) = mv_1 + mv_2$	5	
NEL $\quad v_1 - v_2 = -\frac{1}{3}(u\cos\alpha - 0)$	5	
$v_1 = \dfrac{u\cos\alpha}{3}$	5	
$\tan(\alpha+\theta) = \dfrac{u\sin\alpha}{v_1}$	5	
$\dfrac{\tan\alpha+\tan\theta}{1-\tan\alpha\tan\theta} = \dfrac{3u\sin\alpha}{u\cos\alpha}$		
$= 3\tan\alpha$		
$\tan\alpha + \tan\theta = 3\tan\alpha - 3\tan^2\alpha\tan\theta$		
$\tan\theta = \dfrac{2\tan\alpha}{1+3\tan^2\alpha}$	5	25

6. **(a)** A particle of mass 0·5 kg is suspended from a fixed point P by a spring which executes simple harmonic motion with amplitude 0·2 m.
The period of the motion is 2 seconds.

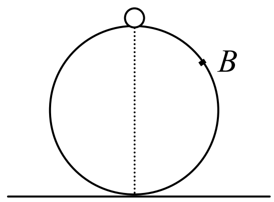
Find **(i)** the maximum acceleration of the particle

(ii) the greatest force exerted by the spring correct to one place of decimals.

(i)	$\frac{2\pi}{\omega} = 2$ $\Rightarrow \omega = \pi$	5	
	$a = \omega^2 A$ $= \pi^2 \times 0{\cdot}2$ $= \frac{\pi^2}{5}$	5	
(ii)	$F = m \times a$ $T - mg = 0{\cdot}5 \times \frac{\pi^2}{5}$	5	
	$T = \frac{g}{2} + \frac{\pi^2}{10}$ $= 5{\cdot}9$ N	5	20

6 **(b)** A particle of mass m kg lies on the top of a smooth fixed sphere of radius 30 cm.

The particle is slightly displaced and slides down the sphere. The particle leaves the sphere at B.



(i) Find the speed of the particle at B.

(ii) The horizontal distance, in metres, of the particle from the centre of the sphere t seconds after it has left the surface of the sphere is $\frac{\sqrt{5}}{10}+kt$. Find the value of k correct to two places of decimals.

(i)

$$\tfrac{1}{2}mv^2 = mg\left(0{\cdot}3-0{\cdot}3\cos\alpha\right)$$
$$v^2 = 0{\cdot}6g\left(1-\cos\alpha\right) \qquad 5$$

$$mg\cos\alpha - R = \frac{mv^2}{0{\cdot}3}$$
$$R=0 \Rightarrow v^2 = 0{\cdot}3g\cos\alpha \qquad 5$$

$$0{\cdot}3g\cos\alpha = 0{\cdot}6g\left(1-\cos\alpha\right)$$
$$\cos\alpha = 2\left(1-\cos\alpha\right)$$
$$\cos\alpha = \frac{2}{3} \qquad 5$$

$$v^2 = 0{\cdot}3g\cos\alpha = 0{\cdot}2g$$
$$\Rightarrow v = 1{\cdot}4 \text{ m s}^{-1} \qquad 5$$

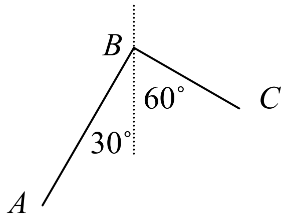
(ii)

$$x = 0{\cdot}3\sin\alpha + 1{\cdot}4\cos\alpha\times t \qquad 5$$
$$= 0{\cdot}3\times\frac{\sqrt{5}}{3}+1{\cdot}4\times\frac{2}{3}\times t$$
$$= \frac{\sqrt{5}}{10}+\frac{2{\cdot}8}{3}\times t$$
$$\Rightarrow k = \frac{14}{15} = 0{\cdot}93 \qquad 5$$

30

7. **(a)** A uniform wire ABC is bent at right angles at B. When it is suspended from B the parts AB and BC make angles of 30° and 60° respectively with the vertical.

If the mass per unit length of the wire is m and $|AB| = h|BC|$ find the value of h.



$$mgh|BC| \times \tfrac{1}{2}h|BC|\sin 30 =$$ 5, 5

$$mg|BC| \times \tfrac{1}{2}|BC|\sin 60$$ 5

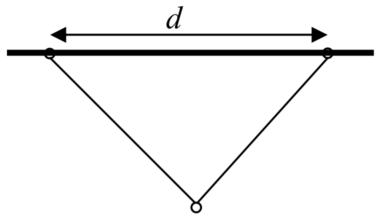
$$h^2 \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$h = \sqrt{\sqrt{3}}$$

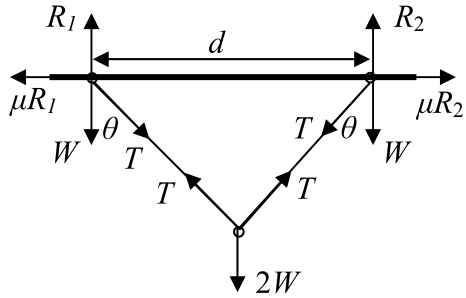
$$= 1{\cdot}316$$ 5

20

7. **(b)** Two rough rings of equal weight W are a distance d apart on a horizontal rod.
A light smooth inelastic string of length 2ℓ connects the rings.
Another ring of weight $2W$ slides on the string.
The coefficient of friction between the rough rings and the rod is μ.



Show that the system remains at rest if $d < \dfrac{4\mu\ell}{\sqrt{1+4\mu^2}}$.



$$2\,T\cos\theta = 2W$$ 5
$$T\cos\theta = W$$

$$R_1 = W + T\cos\theta$$ 5
$$= 2W$$

$$R_1\left(\frac{d}{2}\right) = W\left(\frac{d}{2}\right) + \mu R_1\sqrt{\ell^2 - \frac{d^2}{4}}$$ 5, 5

$$2W\left(\frac{d}{2}\right) = W\left(\frac{d}{2}\right) + \mu(2W)\sqrt{\ell^2 - \frac{d^2}{4}}$$ 5

$$d = 4\mu\sqrt{\ell^2 - \frac{d^2}{4}}$$

$$d^2 = 16\mu^2\ell^2 - 4\mu^2 d^2$$

$$d^2\left(1+4\mu^2\right) = 16\mu^2\ell^2$$

$$\Rightarrow d = \frac{4\mu\ell}{\sqrt{1+4\mu^2}}$$

$$\Rightarrow d < \frac{4\mu\ell}{\sqrt{1+4\mu^2}}$$ 5

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8. **(a)** Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

Let $\mathrm{M} =$ mass per unit area

mass of element $= \mathrm{M}\{2\pi x \, \mathrm{d}x\}$

moment of inertia of the element $= \mathrm{M}\{2\pi x \; \mathrm{d}x\}x^2$ 5

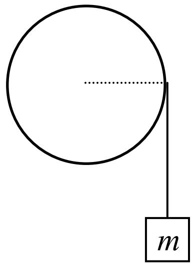
moment of inertia of the disc $= 2\pi \mathrm{M} \int_0^r x^3 \, \mathrm{d}x$ 5

$$= 2\pi \mathrm{M}\left[\frac{x^4}{4}\right]_0^r$$ 5

$$= \mathrm{M}\pi \frac{r^4}{2}$$

$$= \tfrac{1}{2} m r^2$$ 5 20

8. **(b)** A string is wrapped around a smooth pulley wheel of radius r.
A particle of mass m is attached to the string.



The axis of rotation of the wheel is horizontal, perpendicular to the wheel, and passes through the centre of the wheel.

The moment of inertia of the wheel about the axis is I.
The particle is released from rest and moves vertically downwards.

(i) Find, in terms of I, m and r, the tension in the string.

(ii) If the acceleration of the particle is $\frac{g}{5}$, find the mass of the pulley wheel in terms of m.

(i)

$$\tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv^2 = mgh$$

$$\tfrac{1}{2}I\frac{v^2}{r^2} + \tfrac{1}{2}mv^2 = mgh$$ 5

$$v^2 = u^2 + 2fs$$

$$= 0 + 2fh$$ 5

$$\tfrac{1}{2}I\frac{2fh}{r^2} + \tfrac{1}{2}m(2fh) = mgh$$

$$f\left\{\frac{I}{r^2} + m\right\} = mg$$

$$\Rightarrow f = \frac{mg}{\frac{I}{r^2} + m} = \frac{mgr^2}{I + mr^2}$$ 5

$$mg - T = mf$$ 5

$$T = mg - mf$$

$$= mg\left\{1 - \frac{mr^2}{I + mr^2}\right\}$$

$$T = \frac{mgI}{I + mr^2}$$ 5

(ii)

$$f = \frac{mgr^2}{I + mr^2} = \frac{g}{5}$$

$$5mr^2 = I + mr^2$$

$$4mr^2 = \tfrac{1}{2}Mr^2$$

$$\Rightarrow M = 8m$$ 5

30

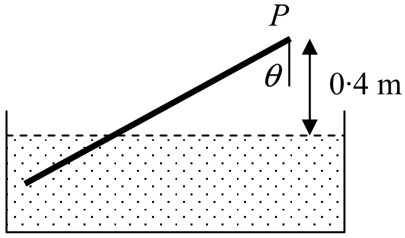
9. **(a)** Stainless steel is an alloy of iron, chromium and nickel. A piece of stainless steel consists of 70% iron, 20% chromium and 10% nickel by volume.

The relative densities of iron, chromium and nickel are 7·8, 7·2 and 8·9 respectively.

Find the relative density of stainless steel.

$m_I + m_C + m_N = m_{SS}$	5	
$7800 \times 0{\cdot}7V + 7200 \times 0{\cdot}2V + 8900 \times 0{\cdot}1V = \rho V$	5, 5	
$\rho = 7790$		
$s = 7{\cdot}79$	5	20

9 **(b)** A uniform rod, of length 2 m and weight W, is freely hinged at a point P.

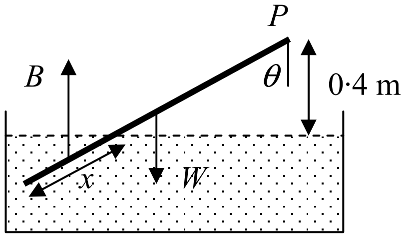


The rod of relative density 0·756 is free to move about a horizontal axis through P. The other end of the rod is immersed in a liquid of relative density 0·9.

The point P is 0·4 m above the surface of the liquid.

The rod is in equilibrium and is inclined at an angle of θ to the vertical.

Find **(i)** the length of the immersed part of the rod
(ii) the value of θ.



(i) $$B = \frac{\frac{x}{2}W \times 0{\cdot}9}{0.756}$$ 5

$$W \times 1\sin\theta = B \times \left(2 - \frac{x}{2}\right)\sin\theta$$ 5, 5

$$W = \frac{\frac{x}{2}W \times 0{\cdot}9}{0{\cdot}756}\left(2 - \frac{x}{2}\right)$$

$$1{\cdot}68 = x\left(2 - \frac{x}{2}\right)$$ 5

$$x^2 - 4x + 3{\cdot}36 = 0$$

$$\Rightarrow x = 1{\cdot}2 \text{ m.}$$ 5

(ii) $$\cos\theta = \frac{0{\cdot}4}{2 - 1{\cdot}2} = \frac{1}{2}$$

$$\theta = 60^\circ$$ 5

30

10. (a) Newton's law of cooling states that 'the rate of cooling of a body is proportional to the difference between the temperature of a body and the temperature of its surroundings.' If θ is the difference between the temperature of a body and the temperature of its surroundings then

$$\frac{d\theta}{dt} = -k\theta.$$

A body cools from 80˚ C to 60˚ C in 10 minutes. The temperature of the surroundings is maintained at 20˚ C. Find

(i) the value of k

(ii) the temperature of the body after a further 15 minutes.

(i)

$$\frac{d\theta}{dt} = -k\theta$$

$$\int \frac{1}{\theta}\, d\theta = \int -k\, dt \qquad 5$$

$$\int_{60}^{40} \frac{1}{\theta}\, d\theta = -k\int_{0}^{10} dt$$

$$\left[\ln\theta\right]_{60}^{40} = \left[-kt\right]_{0}^{10} \qquad 5$$

$$\ln 40 - \ln 60 = -10k$$

$$10k = \ln\frac{60}{40}$$

$$\Rightarrow \quad k = \frac{1}{10}\ln 1{\cdot}5 = 0{\cdot}0405 \qquad 5$$

(ii)

$$\left[\ln\theta\right]_{40}^{\theta-20} = \left[-kt\right]_{0}^{15} \qquad 5$$

$$\ln\frac{\theta-20}{40} = -15k$$

$$\frac{\theta-20}{40} = e^{-0{\cdot}6075}$$

$$\theta - 20 = 21{\cdot}7884$$

$$\theta = 41{\cdot}8^{\circ} \qquad 5$$

25

10 **(b)** A particle of mass m is fired horizontally through a block of resistive gel. The resistance to motion is mkv^2 N when v m s^{-1} is the speed. The particle enters the gel at a speed of 1000 m s^{-1} and $\frac{1}{100}$ seconds later exits the gel at a speed of 10 m s^{-1}.

(i) Show that $k = \frac{99}{10}$.

(ii) Use the fact that $\int \frac{dt}{9900t+1} = \frac{1}{9900}\ln(9900t+1) + C$ or otherwise to show that the length of the block of gel is $\frac{10}{99}\ln 100$ m.

(*i*)

$$\frac{dv}{dt} = -kv^2 \qquad 5$$

$$\int_{1000}^{10} \frac{1}{v^2}\,dv = -k\int_0^{0{\cdot}01} dt \qquad 5$$

$$\left[-\frac{1}{v}\right]_{1000}^{10} = \left[-kt\right]_0^{0{\cdot}1} \qquad 5$$

$$\frac{-1}{10} + \frac{1}{1000} = -k \times 0{\cdot}01$$

$$\Rightarrow k = \frac{99}{10} \qquad 5$$

(*ii*)

$$v\frac{dv}{dx} = -kv^2$$

$$= -\frac{99}{10}v^2$$

$$\int_{1000}^{10} \frac{1}{v}\,dv = -\frac{99}{10}\int_0^x dx$$

$$\left[\ln v\right]_{1000}^{10} = \left[-\frac{99x}{10}\right]_0^x$$

$$\ln\frac{10}{1000} = -\frac{99x}{10}$$

$$\Rightarrow x = \frac{10}{99}\ln 100 \qquad 5 \qquad 25$$